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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2018

First Semester

Faculty of Science

Branch I (a)—Mathematics

MT 01 C01—LINEAR ALGEBRA

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Are the vectors $(1, 8, -7, 3), (-2, 1, 5, 2), (3, -1, 4, 1), (5, 0, 1, 3)$ linearly independent in $\mathbb{R}^4$.

2. Find three vectors in $\mathbb{R}^3$ which are linearly dependent, and lies on a two dimesional subspace.

3. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T : V \to V$ defined by $T(A) = AB - BA$ linear operator and, if f is the trace function then what is $T^t f$?

4. Find two linear operators T and U on $\mathbb{R}^2$ such that $TU = 0$ but $UT \neq 0$.

5. Let K be a commutative ring with identity and let n be a positive integer. Show that there exist at least one determinant function on $K^{n \times n}$.

6. Check whether the function D on the set of 3×3 matrices over the field of real numbers defined by $D(A) = A_{11} + A_{22} + A_{33}$ linear.

7. Show that similar matrices have the same characteristic polynomial.

8. Show that every matrix A such that $A^2 - A$ is similar to a diagonal matrix.

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Let $\alpha = (x_1, x_2)$, and $\beta = (y_1, y_2)$ be vectors in $\mathbb{R}^2$ such that $x_1y_1 + x_2y_2 = 0, x_1^2 + x_2^2 = y_1^2 + y_2^2 = 1$. Prove that $\{\alpha, \beta\}$ form a basis for $\mathbb{R}^2$.

10. Let W_1 and W_2 are subspaces of a finite-dimensional vectorspace V. Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.

11. If A and B are $n \times n$ matrices over the field F, Show that $\text{trace}(AB) = \text{trace}(BA)$, also show that similar matrices have same trace.

12. Let A be any $n \times n$ matrix over the field F. Prove that row rank of A is equal to the column rank of A.

13. Let K be a commtative ring with identity, and let A and B be $n \times n$ matrices over K. Prove that $\det(AB) = (\det A)(\det B)$.

14. Let T be a linear operator on the n–dimensional vector space V, and has distinct characteristic values. Prove that T is diagonalizable.

15. Let a, b and c be elements of a field F and $A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$ be the matrix over F. Prove that the characteristic polynomial and minimal polynomial for A is $x^3 - ax^2 - bx - c$.

16. Suppose that $f = (x - c_1)^{d1} \ldots\ldots (x - ck)^{dk}$ is the characteristic polynomial of an $n \times n$ matrix A. Show that trace $A = c_1d_1 + c_2d_2 + \ldots\ldots + c_kd_k$.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Let V be a vector space over a subfield of complex numbers. Suppose α, β and γ are linearly independent vectors in V. Prove that $(\alpha+\beta), (\beta+\gamma)$, and $(\gamma+\alpha)$ are linearly independent.

18. Show that every hyperspace in a vector space V is the null space of some non-zero linear functional on V.

19. Let A be an $m \times n$ matrix with real entries. Prove that $A = 0$ if and only if trace $(A^t A) = 0$.

20. Let K be a commutative ring with identity and let n be a positive integer. Show that there is precisely one determinant function on the set of $n \times n$ matrices over K.

21. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial for T is of the form $p = (x - c_1) \ldots (x - c_k)$, where $c_1, \ldots c_k$ are distinct elements of F.

22. Let V be a real vector space and E a projection. Prove that $(I + E)$ is invertible. Find its inverse.

$(3 \times 5 = 15)$